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MODELLING THE AGGREGATE DEMAND FOR ELECTRICITY: SIMPLICITY VS. VIRTUOSITY

by

Ernst R. Berndt*

WP# 1415-83 March 1983

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I. INTRODUCTION

The analysis of factors affecting demand for electricity has become an important topic of econometric research recently, due partly to rapidly increasing electricity prices since 1973, smaller than expected growth rates of demand, and continuing controversies regarding pricing policy, future capital needs and construction plans within the electric utility industry. Econometric research has focused particular attention on estimates of electricity income and price elasticities. This econometric literature has been surveyed by, among others, Lester Taylor [1975], Raymond S. Hartman [1979], and Douglas R. Bohi [1981].

Taylor's survey is particularly striking, for in addition to castigating almost all of the existing empirical literature, it outlines clear directions for future research. More specifically, Taylor shows that on the basis of economic theory, both average intramarginal and marginal price should appear as regressors in the demand equation. He then goes on to assert (somewhat mistakenly) that up to 1975 not one study had included both these variables as regressors, and that "...no amount of econometric virtuosity can overcome the problems caused by the failure to correctly specify the price of electricity in the demand function."[1] Taylor recommends as the first order of business, the rather costly construction of a data set for prices based on actual rate schedules. Results of such data construction and model estimation have since been published by Taylor et al. [1977].

While Taylor's argument is certainly plausible, the cost-conscious researcher might ask, "So what?" Hence, in Section II of this note I show

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analytically that the quantitative, empirical significance of Taylor's omitted variable argument is negligible, since least squares estimates with the average intramarginal price variable included as a regressor will typically be virtually identical to least squares estimates with that variable excluded. In Section III, I illustrate my analytical remarks empirically using data from H.S. Houthakker's pioneering [1951b] study, and also amend considerably Taylor's interpretation of Houthakker's article. In section IV, I present brief concluding remarks.

II. ECONOMETRIC CONSEQUENCES OF INCORRECTLY OMITTING THE INTRAMARGINAL AVERAGE PRICE VARIABLE

A basic thrust of Taylor's survey is that it is necessary to include as regressors in the residential electricity demand equation both the marginal ("tailing block" or "running") price of electricity faced by the "typical customer" and the average price per kWh of electricity consumed up to but not including the final block. Alternatively, in place of this average price one can employ total expenditure on electricity up to the final intramarginal block. The qualitative effect of incorrectly omitting the average price or intramarginal expenditure variable is, according to Taylor, as follows:

> If average and marginal prices are positively correlated (as is likely to be the case), then use of one of the prices in absence of the other will lead, in general, to an upward bias in the estimate of the price elasticity. That this is so follows from the theorem on the impact of an omitted variable."[2]

Taylor does not speculate on the empirical significance of such a misspecification. I now focus attention on that issue.

Let the correct demand equation be of the form

$$(1) \qquad y_i = \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \ldots + \beta_k x_{ki} + u_i \ , \quad i = 1,\ldots,m$$

where for the i^{th} observation y_i is the consumption of electricity per household in kWh, x_{1i} is the marginal price of electricity, x_{2i} is the intramarginal average price of electricity (i.e., the average price up to but not including the final block of the typical consumer), $x_{3i},x_{4i},\ldots,x_{ki}$ are other explanatory variables such as income, cooling and/or heating degree days, female labor force participation, lagged electricity consumption per household, etc. and u_i is the random disturbance. Without loss of generality, I measure all variables in terms of deviations from their means; this implies that no intercept term appears in (1).

Let the <u>estimated</u> least squares regression equation when all k regressors are included be (deleting i subscripts)

$$(2) \qquad \hat{y} = b_{y1.k}x_1 + b_{y2.k}x_2 + b_{y3.k}x_3 + \ldots + b_{yk.k}x_k$$

where $\hat{y}$ is the least squares "fitted" or "predicted" value of y, and where $b_{y1.k},b_{y2.k},\ldots,b_{yk.k}$ are the least squares coefficients on $x_1,\ldots,x_k$ (in the presence of all k regressors). Hereafter (2) is called the "correct" regression equation.

The misspecification noted by Taylor occurs when x_2 is incorrectly omitted from the regression equation (2). Let the misspecified least squares

regression equation with x_2 omitted be

$$(3) \qquad \hat{y} = b_{y1}x_1 + b_{y3}x_3 + \ldots + b_{yk}x_k$$

where $b_{y1}, b_{y3}, \ldots, b_{yk}$ are the least squares coefficients on $x_1, x_3, \ldots, x_k$ in the misspecified regression equation. Taylor notes that (3) is the equation fitted by H.S. Houthakker (1951b); even though Houthakker correctly included the marginal price variable x_1, according to Taylor, Houthakker incorrectly excluded x_2.

The econometric consequences of this misspecification on estimates of the marginal price elasticity can be assessed by determining the difference between $b_{y1.k}$ and b_{y1}. Arthur S. Goldberger [3] [1968, Chapter 3] has derived the analytical relationship between the least squares estimates $b_{y1.k}$ and b_{y1}:

$$(4) \qquad b_{y1.k} = b_{y1} - b_{21}b_{y2.k}$$

where b_{21} is the least squares coefficient in the "auxiliary" regression equation

$$(5) \qquad \hat{x}_2 = b_{21}x_1 + b_{23}x_3 + \ldots + b_{2k}x_k \quad .$$

In the present context, b_{21} is the least squares coefficient when intramarginal average price is regressed on marginal price and the other regressors in (3).

Equation (4) allows easy determination of the qualitative consequences of incorrectly omitting x_2. Assuming that b_{y1}, $b_{y1.k}$ and $b_{y2.k}$ are

negative (as is suggested by economic theory) and that b_{21} is positive, then b_{y1} is larger in absolute value than $b_{y1.k}$ and b_{y1} provides an absolute upward biased estimate of the marginal price elasticity. Notice, however, that from (4) the sign of the bias depends not only on the sign of b_{21}, but also on the sign of $b_{y2.k}$.

A more precise quantitative assessment of this bias is also possible. Since Taylor's preferred specifications typically involve double-logarithmic regressions, let y and each of the x's in (1), (2), and (3) be logarithmic transforms of the original variables. Under such logarithmic transformations, $b_{y1.k}$ is an estimate of the marginal price elasticity of demand for electricity, and $b_{y2.k}$ is the least squares estimate of the intramarginal average price elasticity of demand. Fortunately, microeconomic theory provides a clear interpretation of $b_{y2.k}$. Since a change in x_2 represents only an income effect and no substitution effect (see Taylor [1975, pp. 75-80] for discussion), $b_{y2.k}$ is the negative of the income elasticity of demand for electricity times the budget share of intramarginal electricity expenditure in the total income of the typical residential customer. Taylor [1975] and Taylor et al. [1977] suggest that a reasonable estimate of the income elasticity is unity, while in Taylor et al. [1977] it is reported that the mean intramarginal budget share by state over the 1961-72 time period is about 0.01.[4] Hence, if the theory and specification were correct, a reasonable value for $b_{y2.k} = -(1.0)\ (.01) = -.01$. Such a value can be inserted into (4).

In order to complete a quantitative assessment of misspecification, estimates of b_{21} must be obtained and inserted into (4). Recall that in the logarithmic context, b_{21} in (5) represents an elasticity of average intramarginal price with respect to marginal price. Although b_{21} is likely

to be sample dependent, it would seem reasonable to expect that utilities with relatively high marginal prices also have relatively large fixed and intramarginal charges in their rate structure, and thus that b_{21} might be positive and perhaps in the range of 0.25 to 1.0. Data for four Alberta electric utilities cover the 1962-76 time period (see Data Metrics, Ltd. [1978]) suggest a mean value of b_{21} of around 0.50. If $b_{y2.k} = -0.01$ and $b_{21} = 0.50$ is inserted into (4), one finds that the bias due to misspecification is

$$(6) \qquad b_{y1.k} - b_{y1} = 0.005 \quad .$$

This is a rather strong and significant result, for it suggests that if one incorrectly omits the intramarginal average price variable, then the "correct" and the "misspecified" least squares estimates of the marginal price elasticity are virtually identical. For example, using the "correct" Taylor et al. [1977] $b_{y1.k}$ estimate of -0.8, one finds that the "incorrect" estimate would be virtually identical at -0.805. Hence the cost of this misspecification appears to be very small--especially if viewed relative to typical standard error estimates and to costs of collecting intramarginal expenditure data.

The actual difference between $b_{y1.k}$ and b_{y1} will of course depend on the sample values of $b_{y2.k}$ and b_{21}. Assuming that income elasticities of demand for electricity equal unity, in Table 1 I present differences between the "correct" estimates $b_{y1.k}$ and the "misspecified" estimates b_{y1} under alternative values for the intramarginal budget shares and b_{21}. The essential point is clear: the empirical significance of incorrectly omitting x_2 appears to be negligible.

TABLE 1

Differences Between "Correct" Estimates $b_{y1.k}$ and "Misspecified" Estimates b_{y1} Under Alternative Assumptions

Intramarginal Budget Shares	Values of b_{21} .25	.50	.75	1.00
.005	.00125	.00250	.00375	.00500
.010	.00250	.00500	.00750	.01000
.025	.00625	.01250	.01875	.02500
.050	.01250	.02500	.03750	.05000

The above results are not affected substantially if average intramarginal price were replaced by intramarginal expenditure; the principal alteration would be the new interpretation of b_{21} as the elasticity of intramarginal expenditure with respect to marginal price.[5]

Although the above discussion has focussed attention on the omitted variable bias affecting the marginal price elasticity estimate, it is clear that omitting x_2 also affects other coefficients. Fortunately, the bias on other coefficients (such as income elasticities) will also be small. To see this, note that the difference between the "correct" estimate of the coefficient on x_j (denoted by $b_{yj.k}$) and the "misspecified" estimate on x_j due to incorrectly omitting x_2 (denoted by b_{yj}) is

$$(7) \qquad b_{yj.k} - b_{yj} = -b_{2j}b_{y2.k} \quad , \qquad j = 1,3,4\ldots,k$$

where b_{2j} is the least squares coefficient in the regression equation

$$(8) \qquad \hat{x}_2 = \sum b_{2j}x_j \quad , \qquad j = 1,3,4,\ldots,k \quad .$$

Note that $b_{y2.k}$ always appears in (7) regardless of which regression coefficient is being checked for bias. Since it has been shown above that it is reasonable to expect $b_{y2.k}$ to be small (around -0.01 if income elasticities are 1.0 and mean intramarginal budget shares are 0.01), it follows that unless the b_{2j} elasticity is very large, in general the difference between $b_{yj.k}$ and b_{yj} will be rather small.

Taylor's discussion of this last point tends to be less precise and at times mistaken, since he focusses only on b_{2j} and ignores the important role of the small and negative $b_{y2.k}$ coefficient. For example,

> "The extent to which biases will exist depends on the correlation between the variable that is left out and the variables that are included. If, for example, marginal and average price are positively correlated (which is likely) and both are positively correlated with income (which is less likely), then the exclusion of one of the prices will lead to an upward bias in the coefficient for the other price and also to an upward bias in the coefficients for income."[6]

Taylor's comment on the upward bias of the income coefficient is a mistaken conjecture. If one denotes the income variable as x_3 and follows Taylor's assumptions that $b_{y3.k}$, b_{y3} and b_{23} are positive, then from (7) it can be seen that $b_{y3.k} - b_{y3}$ is positive, which implies that omitting x_2 leads to a downward (not upward) bias in the coefficient for income. Note, however, that since $b_{y2.k}$ is relatively small, this downward bias on the income coefficient is likely to be rather minor.

In conclusion, the above analysis suggests that the empirical consequences of incorrectly omitting the average intramarginal price (or intramarginal expenditure) variable from the residential electricity demand equation appear to be very small and negligible. If the regression analyst chooses to omit x_2 (say, because of substantial data gathering costs), he/she can still obtain estimates very close to the "correct" estimates by estimating the

misspecified equation and then inserting reasonable estimates of b_{2j} into (7). While this simple procedure is clearly lacking in "econometric virtuosity",[7] in this case it appears to be more than adequate.

III. EMPIRICAL ILLUSTRATION

The arguments presented above regarding the consequences of misspecification are largely analytical, with occasional reference to "reasonable" parameter values. The practical student of the demand for electricity might well wonder how some of the classic published results would have changed had the regressions been specified in different ways. Thus I now briefly present results of such an analysis.

Fortunately, in the seminal study by Houthakker [1951b], data are presented which allow one to run regressions using various combinations of marginal price, intramarginal expenditure and ex post average price for 42 provincial towns in Great Britain, 1937-1938. Houthakker's study is pioneering in two other aspects:

(i) It appears to be the first published econometric article which reports least squares regression results obtained using an electronic computer. Readers interested in this aspect of intellectual history might consult J. A. C. Brown, H. S. Houthakker and S. J. Prais [1953] for an enlightening and prescient discussion of experiences with the EDSAC (the electronic delay storage automatic calculator at the University Mathematical Laboratory at Cambridge University).

(ii) Houthakker's study clearly recognized the implications of a two-part tariff for electricity, and reports estimates using the marginal price. Moreover, Houthakker states [1951b, Paragraph 1.4.1, p. 367] that he initially ran regressions with marginal price and average fixed charge per customer included as regressors; he chose not to report final results with the latter variable included since "its influence was not statistically significant."[8] Houthakker's analysis thus implemented empirically already in 1951 the theoretical framework discussed by Taylor in his 1975 survey.[9]

In column 1 of Table 2 I reproduce the results reported by Houthakker [1951b]. The dependent variable is (the logarithm of) average electricity consumption per domestic customer on the two-part tariff. To adjust for heteroscedasticity, Houthakker transformed his data, multiplying all observations on each variable by the square root of the number of customers in the town. The notation is as follows: M is money income, P_{-2} and P_{-0} are the marginal ("running") prices of electricity lagged two and zero years, respectively, AP_{-0} is the current ex post average price of electricity, G_{-2} is the two-year lagged marginal price of natural gas, H is the average potential electricity consumption (in kilowatts) of appliances operated by domestic two-part customers in 1937-1938, and F is average fixed charge for domestic two-part tariff customers in 1937-38.[10] As seen in column 1, Houthakker's estimates of the income and price elasticities were 1.166 and -0.893, respectively. Although the potential electricity consumption in kilowatts is a regressor in Houthakker's equation, the fact that the marginal price of electricity is lagged two years prompts Houthakker to interpret these elasticities as long-run estimates.[11]

TABLE 2

ALTERNATIVE DEMAND FOR ELECTRICITY EQUATIONS
USING HOUTHAKKER'S DATA FOR 42 PROVINCIAL TOWNS

Great Britain, 1937-38
(Standard Errors in Parentheses)

Regressor (In Logarithms)	1	2	3	4	5	6	7
Constant	na	-1.053	-0.204	-0.067	-0.230	-0.202	1.891
		(.533)	(.614)	(.679)	(.695)	(.793)	(.492)
M	1.166	1.160	1.065	1.041	1.066	1.062	0.786
	(.088)	(.088)	(.082)	(.096)	(.088)	(.104)	(.081)
P_{-2}	-0.893	-0.879	-0.902	-0.892			
	(.190)	(.189)	(.212)	(.215)			
P_0					-.758	-0.754	
					(.238)	(.248)	
AP_{-0}							-0.912
							(.140)
G_{-2}	0.211	0.208	0.051	0.039	0.079	0.076	0.127
	(.116)	(.118)	(.165)	(.161)	(.189)	(.196)	(.138)
H	0.177	0.176	0.184	0.185	0.194	0.194	0.096
	(.033)	(.033)	(.035)	(.036)	(.038)	(.039)	(.034)
F				0.056		0.010	
				(.113)		(.125)	
R^2	.934	.998	.839	.840	.812	.812	.889

Notes to Table 2:

M is money income, P_{-2} is the marginal electricity price lagged two years, P_{-0} is the current electricity marginal price, AP_{-0} is the ex post average electrticity price in the current period, G_{-2} is the marginal natural gas price lagged two years. H is the average holdings (in kilowatts) of heavy electric equipment bought on hire purchase by domestic two-part customers in 1937-38, and F is average fixed charge for domestic two-part customers in 1937-38.

Column 1 reproduces results as reported by Houthakker [1951b, p. 367], while Column 2 presents my attempt at replication based on Houthakker's published data [1951b, pp. 364-365]. Both columns represent estimates adjusted for heteroscedasticity (i.e., both are "weighted" regressions). Columns 3-7 report various "unweighted" estimates (not adjusted for heteroscedasticity).

na = not available from Houthakker's reported results.

In column 2 of Table 2 I present results of my attempt to replicate Houthakker's empirical findings. As can be seen, the results I obtained using the National Bureau of Economic Research TROLL [1975] program on an IBM 370-Model 168 are very close to the classic ones obtained by Houthakker in 1951 using his own subroutines on the EDSAC electronic computer. This close agreement is remarkable given the small working space available on the EDSAC.[12]

In column 3 of Table 2 I report results using an unweighted regression. The point estimates of the income and own-price elasticities are not affected greatly, although the coefficient estimate and statistical significance of the gas cross-price elasticity is reduced considerably when compared to results of the weighted regression. In column 4 I add the average intramarginal price (average fixed charge) variable F. The coefficient on F is positive and quite large, although a 95% confidence interval based on the rather large standard error estimate would include a theoretically plausible estimate of about -.004 to -.005 (the mean intramarginal expenditure budget share for the 42 towns was about .0044, and the estimated income elasticity is slightly greater than unity). Of particular interest is the fact that the difference between the electricity own-price elasticity estimates in the "correct" (column 4) and the "misspecified" (column 3) equations is quite small--0.010; similar small differences appear for the other regressors.[13]

In columns 5 and 6 of Table 2 I report estimates with the lagged price P_{-2} replaced by the current price P_{-0}; the resulting electricity price elasticities might then be interpreted as short-run elasticities, especially since the electricity consuming stock variable H is included as a regressor. These price elasticity estimates are smaller (in absolute value) than those in columns 3 and 4; however, the values of -0.758 (column 5) and -0.754 (column

6) seem rather large for short-run elasticities. The differences between the electricity own-price elasticity estimates in the "correct" (column 6) and "misspecified" (column 5) equations again is small -- .004.[14] These two comparisons--column 3 with 4 and column 5 with 6--adequately illustrate the analytical argument presented in Section II above, namely, that the econometric consequences of incorrectly omitting the average intramarginal price variable are negligible.

Finally, in column 7 of Table 2 I report estimates using the current ex post average electricity price variable instead of a marginal price. The resulting average electricity own-price elasticity estimate of -0.912 is about 20% larger in absolute value than those based on the current marginal price (-0.754 and -0.758), but coincidentally is very close to the marginal price elasticity estimates using the two-year lagged marginal price (-0.902 and -0.892). However, the estimated income elasticity using average ex post electricity price (0.786) is considerably smaller than the 1.041 - 1.165 income elasticity estimates using marginal price data. Thus the price elasticity estimate bsed on average ex post price appears to capture a portion of the income effect. Coefficient differences based on the average ex post and marginal price data, however, are much greater than the budget share of electricity.[15]

IV. CONCLUDING REMARKS

The principal point made in this paper is that the quantitative or empirical significance of Lester Taylor's interesting argument in defense of including both average intramarginal and marginal price variables in the electricity demand equation is negligible. Specifically, it has been shown, both analytically and empirically, that least squares estimates with the average intramarginal price variable excluded will typically differ very little from regression estimates with that variable included. Moreover, this conclusion is quite robust in that it holds for all the regression coefficients, not just the price coefficient. Only when the cost or budget share of intramarginal electricity expenditures is substantial, or when the income elasticity is very large (much greater than unity), will the difference between regression coefficients become more important. Empirically, I have demonstrated this analytical argument by using Houthakker's classic data from his [1951b] study. A principal implication of this paper is, therefore, that if the regression analyst chooses to omit the intramarginal price variable due to, say, substantial data gathering costs, he/she can still obtain regression estimates very close to the "correct" ones by using (7).

Although the argument developed in this paper has been applied to the case of demand for electricity, it is of course the case that price schedules for numerous other products also exhibit differences between marginal and average levels. Hence the analysis of this paper has implications for modelling their demand as well. One particular example of differences between marginal and average occurs in the labor supply area, where the provisions of the progressive tax code as well as overtime premia generate nonlinear and at times even non-continuous price-quantity relationships. At the aggregate

level of analysis, one might still assume continuity by appealing to the fact that the micro distribution of kinks is sufficiently dispersed so that continuity is a reasonable approximation; this has been argued by, among others, Marie Corio [1981]. At the micro level of detail, however, future research might best be focussed on the specification and estimation of models that explicitly recognize the discrete endogenous nature of the marginal electricity price. Procedures recently introduced by Wales-Woodland [1979] and Burtless-Hausman [1978] in the context of effects of after-tax wage rates on labor supply could be adapted to the electricity demand context. At the present time, relatively little is known regarding the extent to which estimates and inference based on these computationally more costly and sophisticated estimation procedures would differ from those based on simpler estimation techniques.

FOOTNOTES:

[1]Taylor [1975], p. 106.

[2]Taylor [1975], p. 80.

[3]Also see Zvi Griliches [1957] and Henri Theil [1957].

[4]See Taylor et al. [1977], p. 7-4.

[5]Alberta data 1962-1976 for four utilities (see Data Metrics Ltd. [1978]) suggest that a reasonable value of b_{21} in such a context is about 0.75. If one inserts this value into (4), assumes an intramarginal budget share of .01 and an income elasticity of 1.0, then one obtains a bias of .0075.

[6]Taylor [1975], p. 102.

[7]Taylor [1975], p. 106.

[8]Houthakker, [1951b], p. 36. The analytical results of Section II above suggest of course that this coefficient should be very small.

[9]Taylor appears to have overlooked this aspect of Houthakker's study, for in his empirical survey Taylor states: "... not one study has recognized that completely proper treatment of price in this context requires that intramarginal prices as well as the marginal price be represented in the demand function" [1975, p. 102].

[10]The F variable is computed as average total expenditure on electricity by two-part consumers in 1937-38 (in pounds sterling) minus the product of the running charge on domestic two-part tariffs in 1937-38 (in pence per kWh) times average consumption per two-part tariff consumer (in kWh) divided by 240 (since at that time there were 12 pence per shilling and 20 shillings per pound sterling).

[11]Taylor's discussion of Houthakker's results on this issue is somewhat confusing. In particular, Taylor [1975, p. 84] states "Houthakker is silent as to whether the elasticities he has estimated refer to the short run or long run." In Houthakker [1951a], however, reference is made to the Houthakker [1951b] study. Houthakker states clearly [1951a, pp. 18-19] that: "From pre-war information the long-term elasticity of demand with respect to the running charge in a two-part tariff has been estimated at about 0.9". Taylor instead states "... in view of the presence of the holdings of heavy electrical equipment as a predictor ... they should be interpreted ... as short-run elasticities" [1975, p. 84]. While Taylor's short-run interpretation is probably preferable a priori (although the price variable is lagged two years), Houthakker unambiguously interpreted his results as long-run estimates. In any case, the -0.9 price elasticity estimate is closer to conventional econometric estimates of the long run.

FOOTNOTES (Continued)

[12]In informal discussions, Houthakker has informed me that he was particularly worried about the accuracy of his matrix inversion subroutine, since the EDSAC computer had only 480 locations, each location comprised 35 bytes, and the inversion subroutine for a 10 x 10 matrix already occupied 200 locations.

[13]The unweighted auxiliary regression equation was (in logarithms)
$\hat{F} = 2.461 + .429M - .173P_{-2} + .224G_{-2} - .010H.$

[14]The estimated auxiliary regression equation here is (in logarithms)
$\hat{F} = 2.916 + .426M - .460P_{-0} + .347G_{-2} - .014H.$

[15]It might be noted that in the regression results reported in columns 4 and 6 of Table 2, the coefficient on F has not been constrained to equal the theoretical value of the negative of the average fixed charge budget share times the estimated income elasticity. A very simple way of imposing this restriction is to redefine the income variable as income minus the intramarginal expenditure, and then to regress electricity on this redefined income variable and the other regressors (but excluding F).

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